UNIVERSITAT POLITÈCNICA DE CATALUNYA

UNIVERSITÀ DEGLI STUDI DI SIENA

Aigües de Barcelona

DMPC on Barcelona water distribution network

WIDE project

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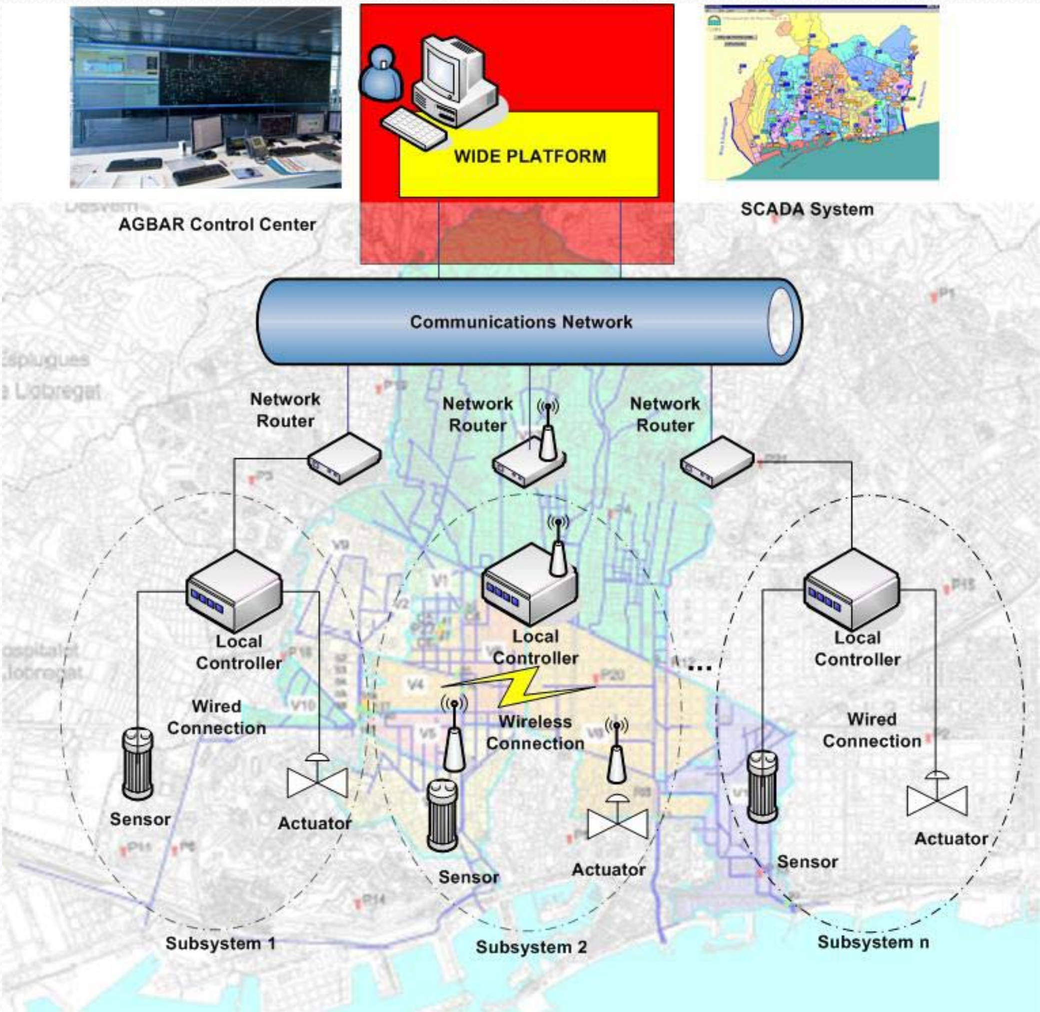
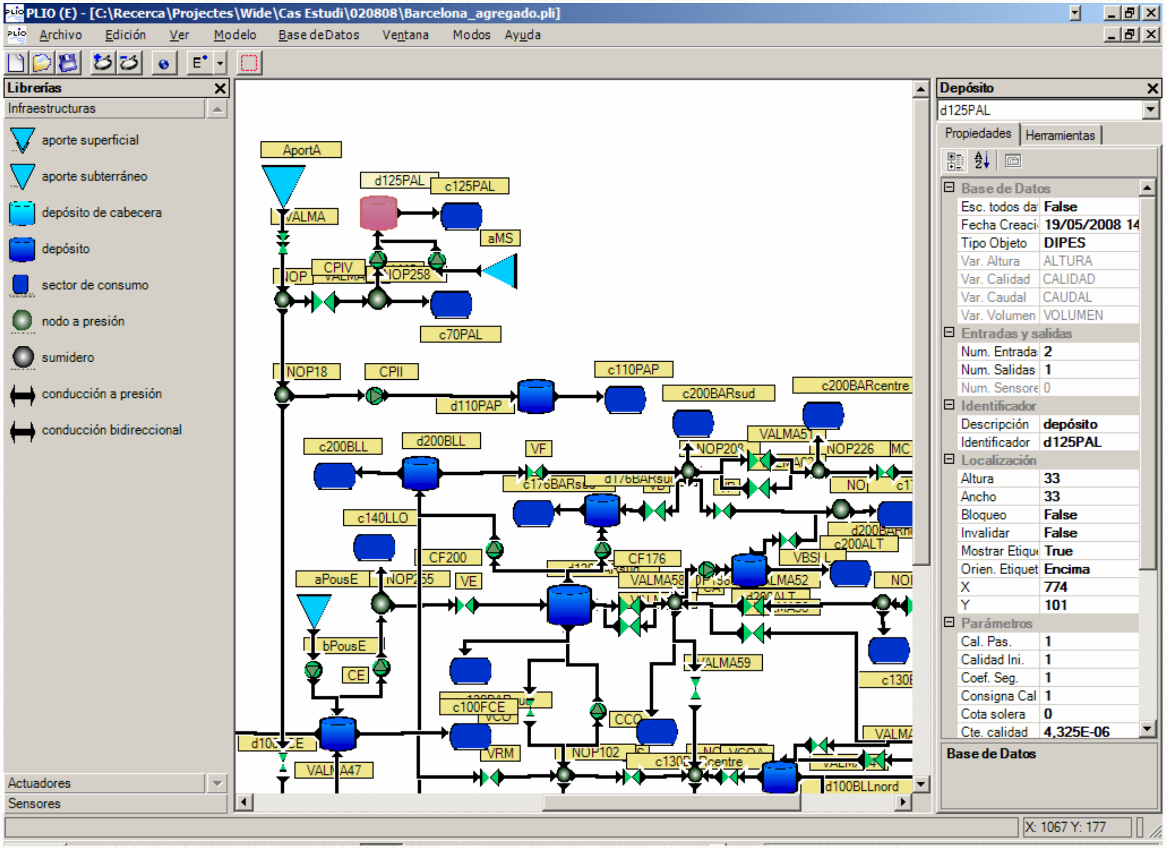
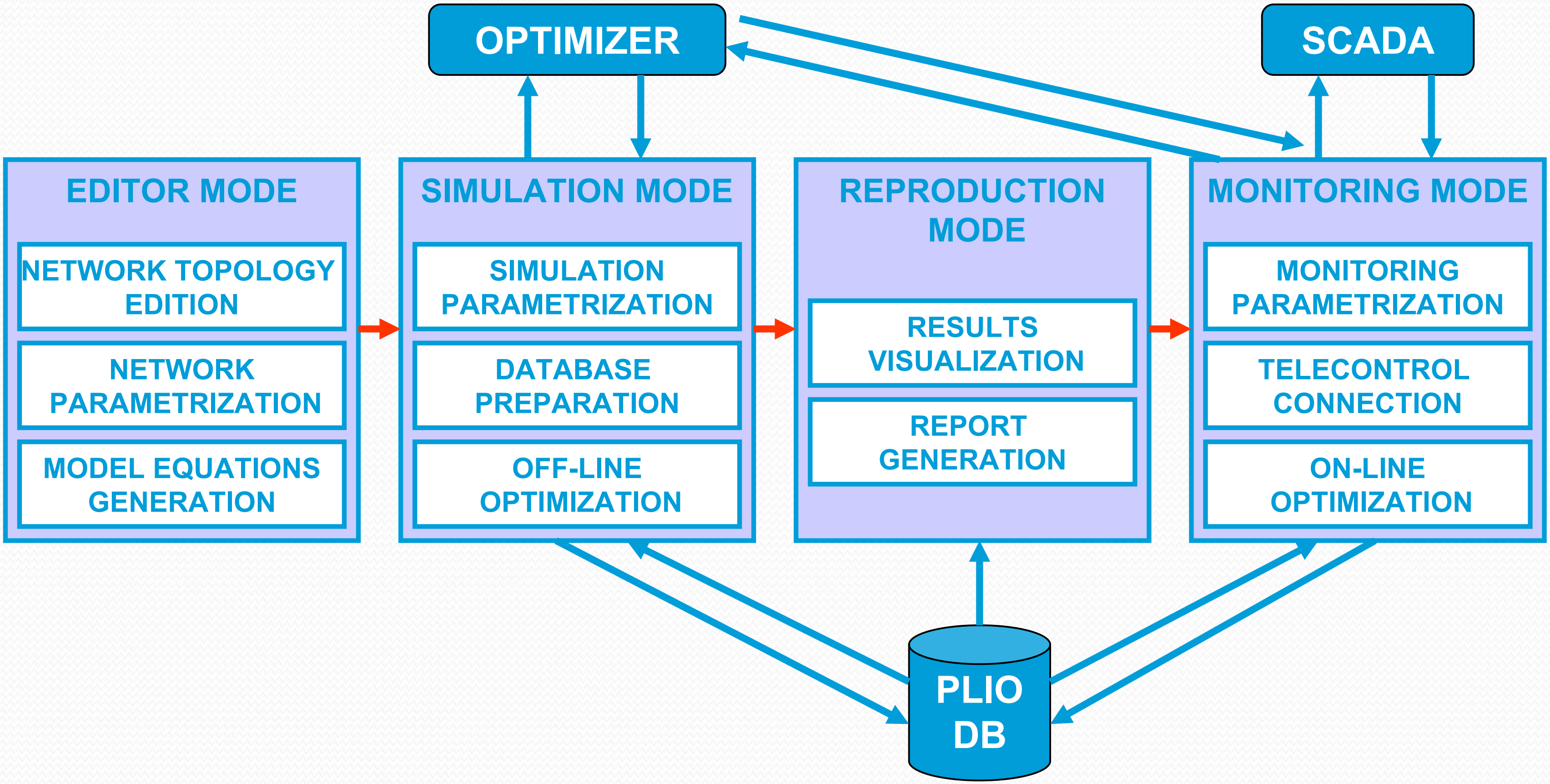
MPC in Water Systems



PLIO: Centralized MPC Controller

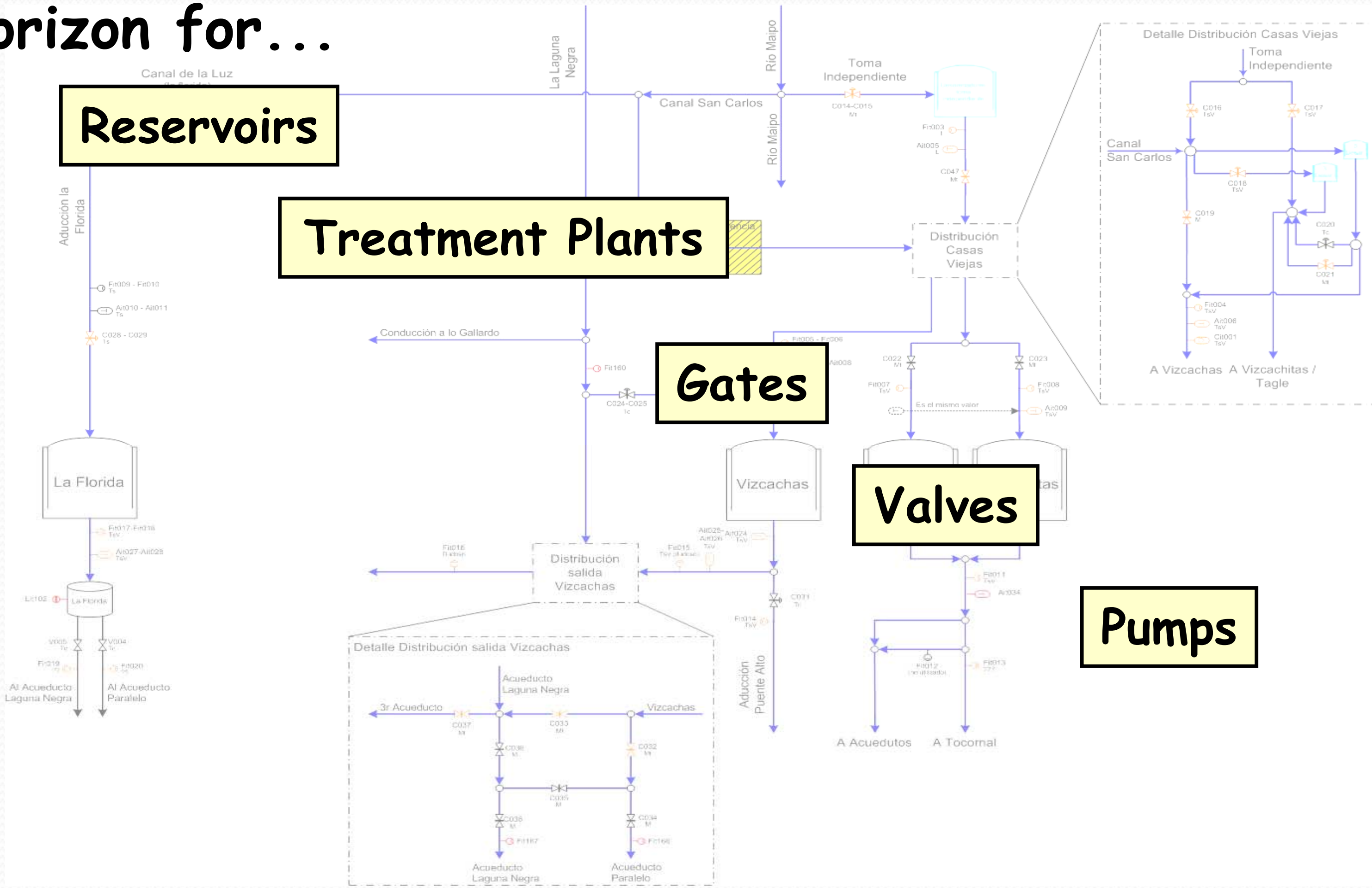


PLIO Architecture



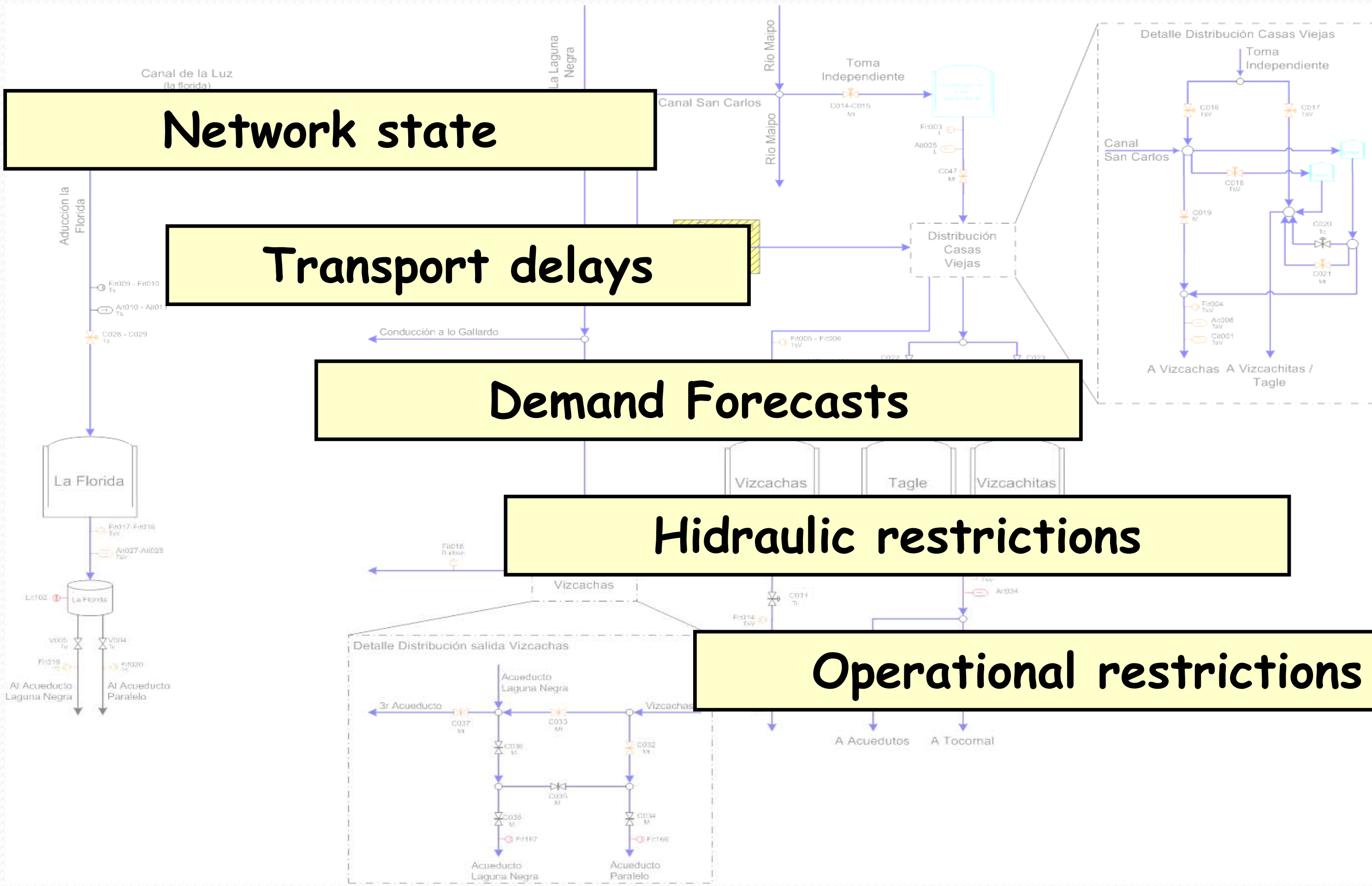
PLIO Operation (1)

Generate optimal control strategies using 24 hour horizon for...



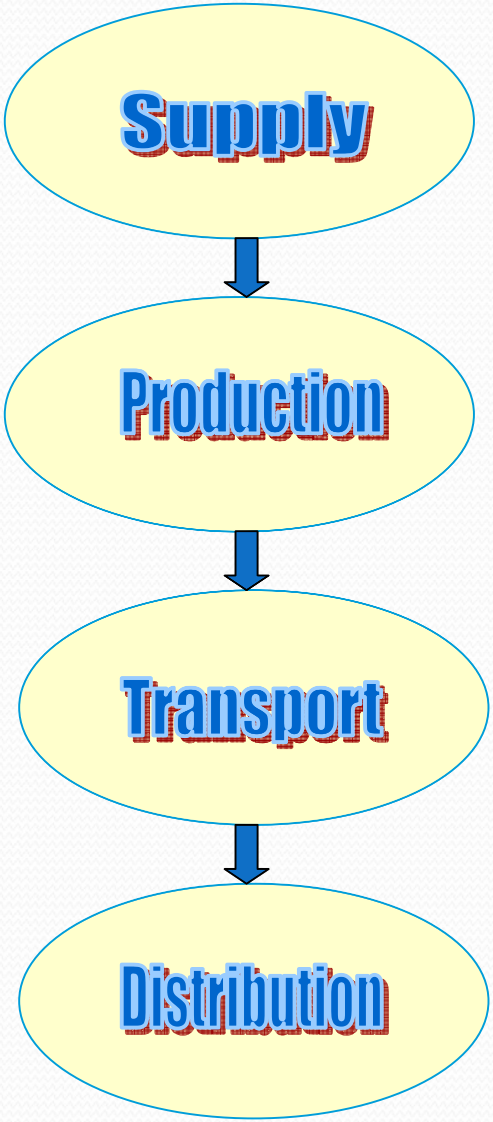
PLIO Operation (2)

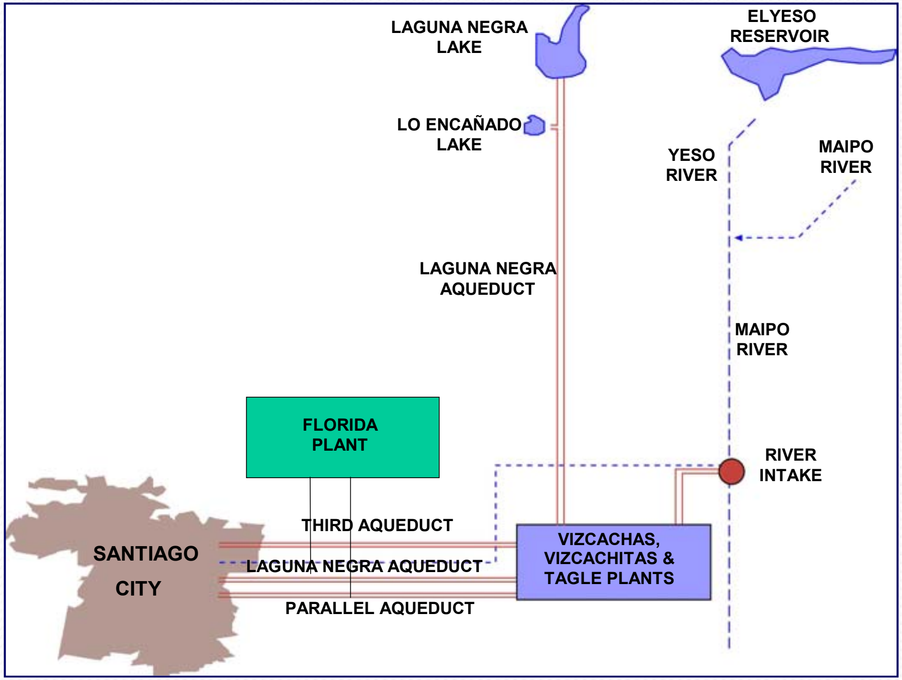
Taking into account...



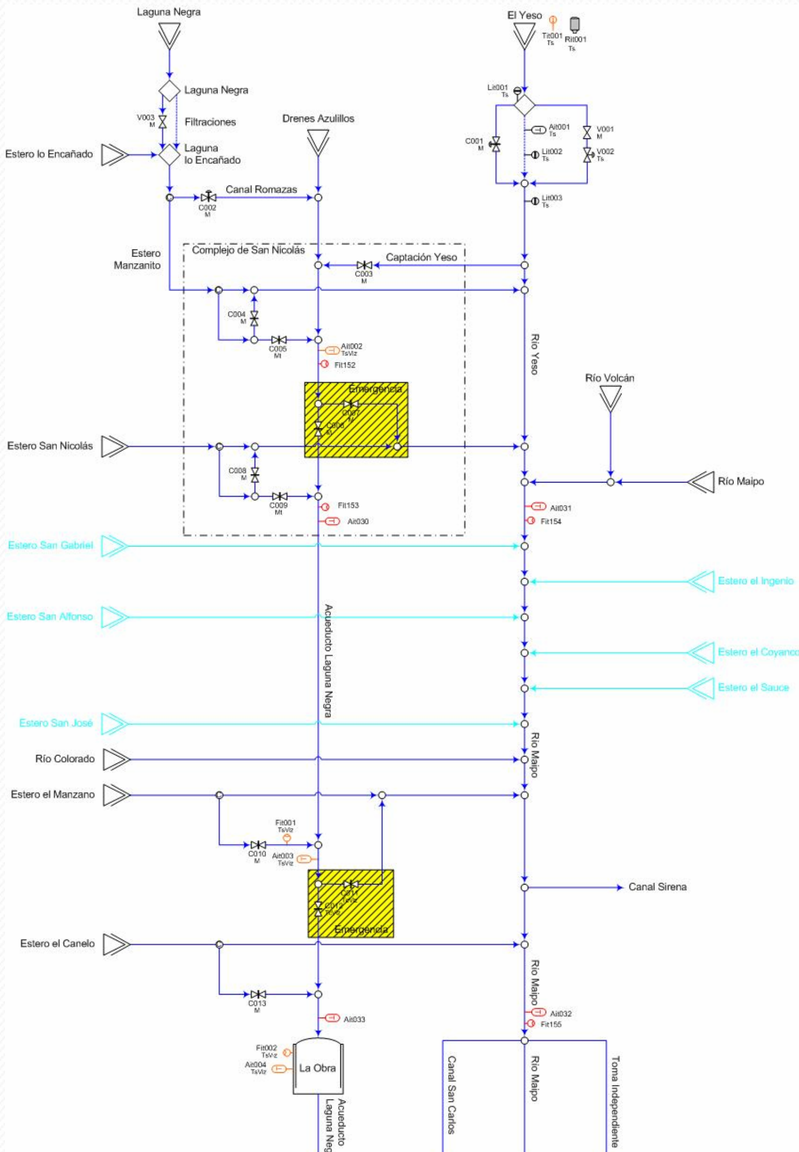
- Network state
- Transport delays
- Demand Forecasts
- Hidraulic restrictions
- Operational restrictions

PLIO real case study: Santiago de Chile

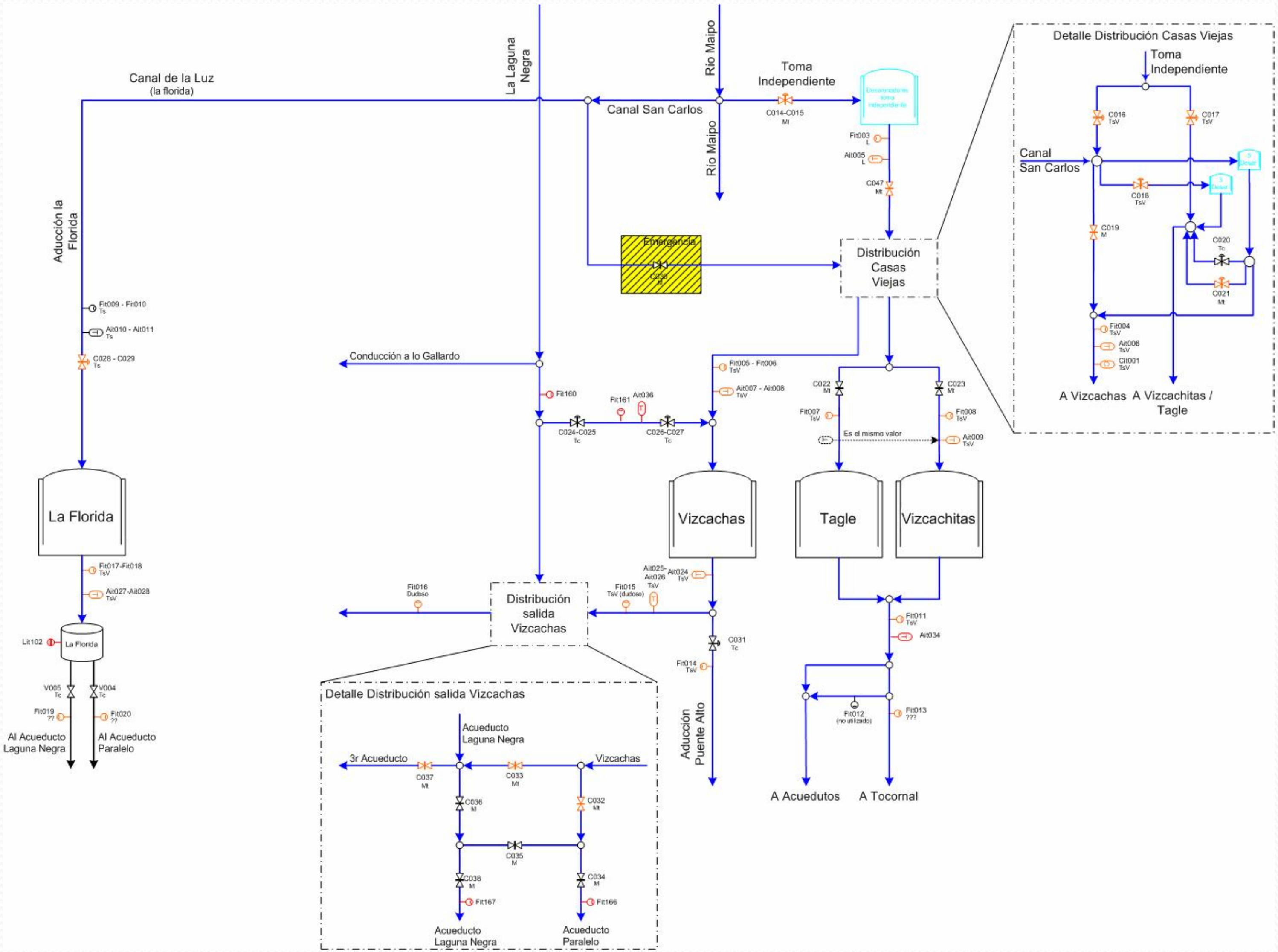




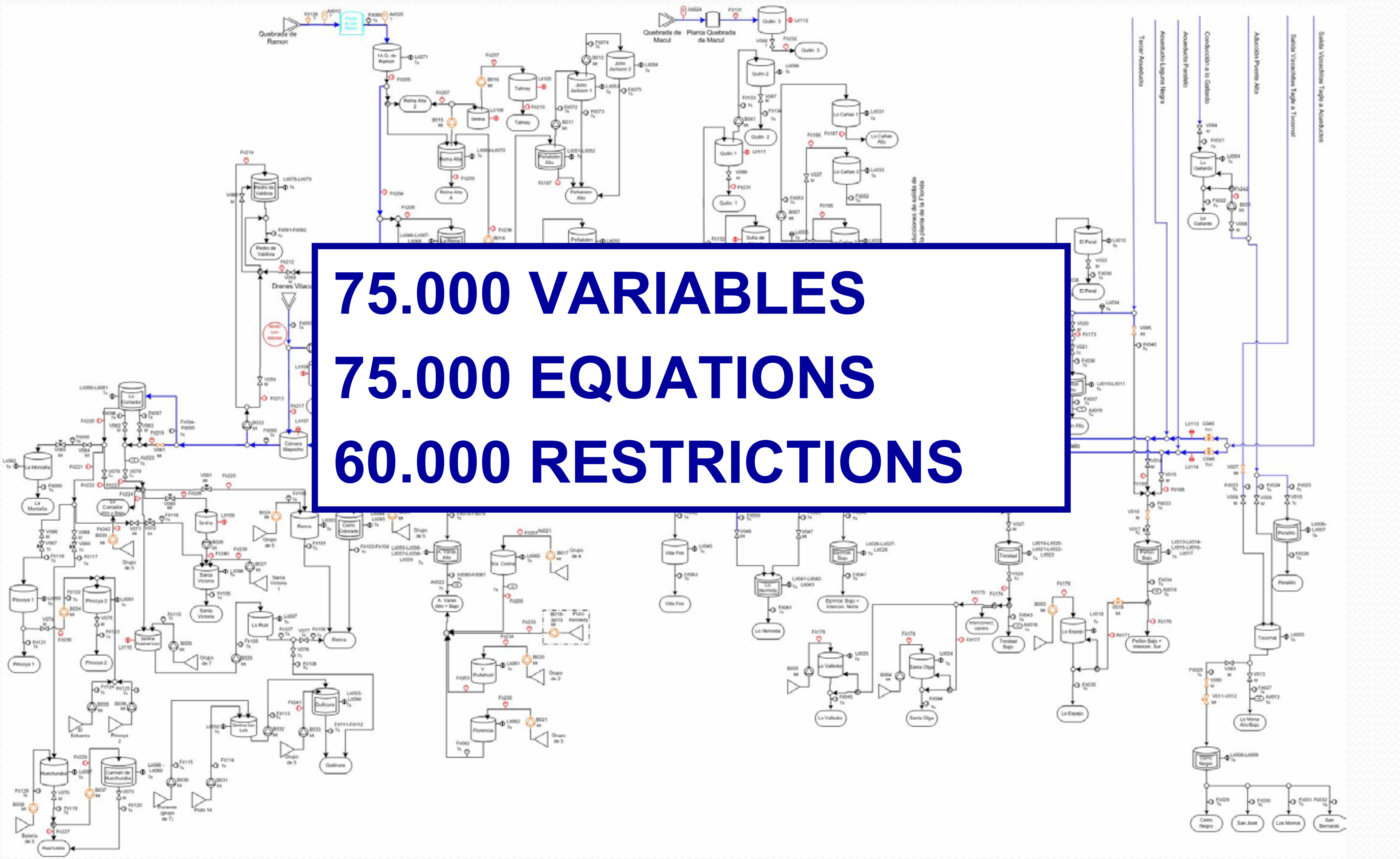
Supply network



Production network



Transport network



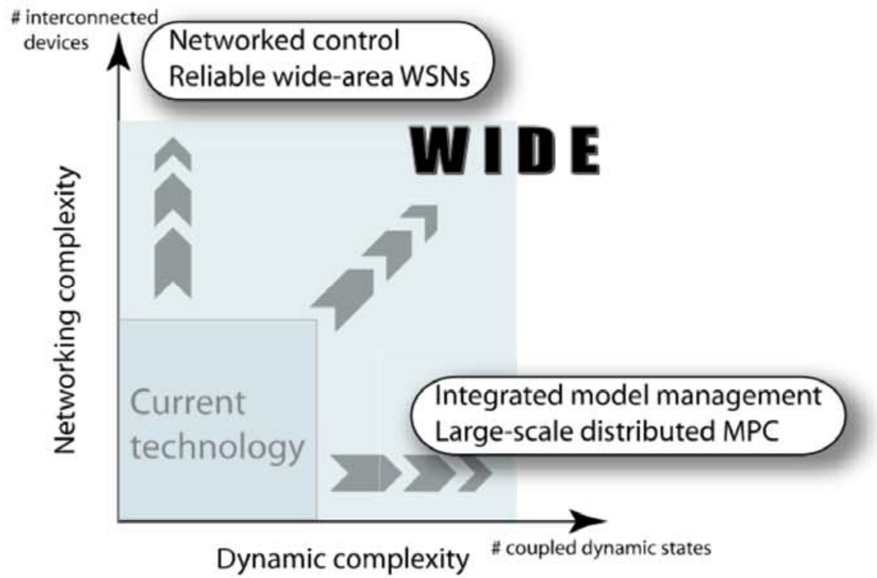
75.000 VARIABLES

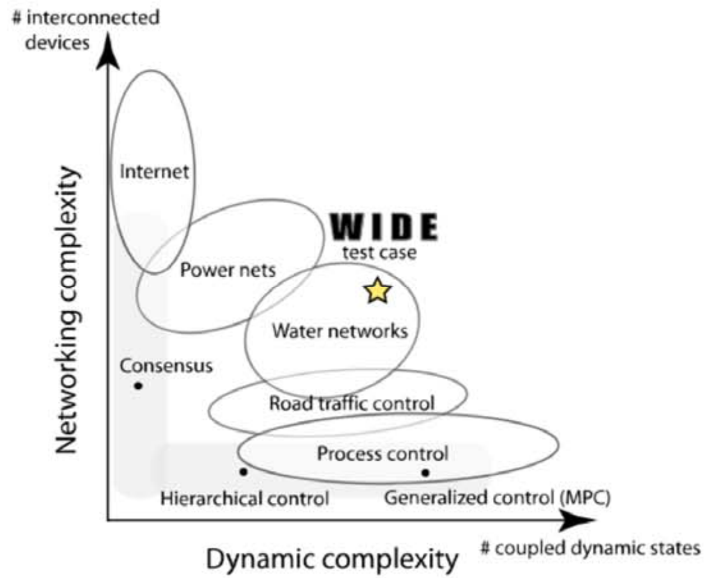
75.000 EQUATIONS

60.000 RESTRICTIONS

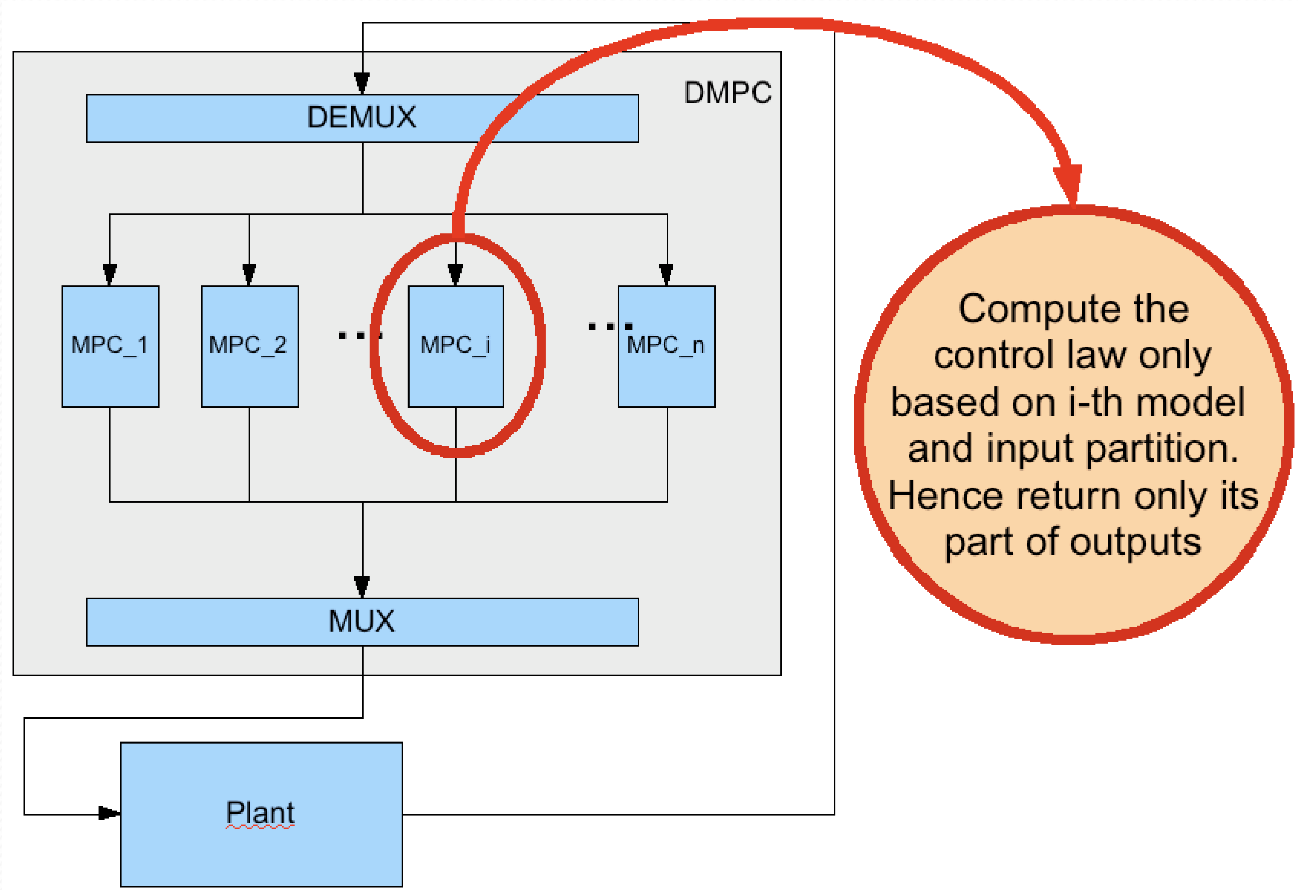
WIDE project

WIDE aims at developing a novel rigorous and integrated framework for advanced control and real-time optimization of large-scale and spatially distributed processes that exploits wireless sensor networks as a pervasive and highly reconfigurable information gathering system, and at validating the approach on a real city water distribution system.





Distributed MPC (1/3)



Distributed MPC (2/3)

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + Q(k) \\ y(k) = Cx(k) \end{cases}$$

$$W_i \in \mathbb{R}^{m \times m_i} \quad \forall i \quad ; \quad Z_i \in \mathbb{R}^{n \times n_i} \quad \forall i$$

$$A_i = W_i^T A W_i \quad ; \quad B_i = W_i^T B Z_i \quad ; \quad C_i = W_i^T C W_i \quad ; \quad Q_i = W_i^T Q$$

$$x_i(k) = W_i^T x(k) \quad ; \quad u_i(k) = Z_i^T u(k)$$

$$\begin{cases} x_i(k+1) = A_i x_i(k) + B_i u_i(k) + Q_i(k) \\ y_i(k) = C_i x(k) \end{cases}$$

Distributed MPC (3/3)

$$W_{tot} = \begin{bmatrix} W_1 & W_2 & \cdots & W_M \end{bmatrix} \quad ; \quad Z_{tot} = \begin{bmatrix} Z_1 & Z_2 & \cdots & Z_M \end{bmatrix}$$

- $w \in W_{tot}, z \in Z_{tot} = [0] \vee [1]$
- each row in both W_{tot} and Z_{tot} must have exactly one element with value 1.
- each column in both W_{tot} and Z_{tot} must have no more than an element with value 1

$$u = \sum_{i=1}^{M} Z_i u_i \qquad\qquad x = \sum_{i=1}^{M} W_i x_i$$

Partitioning methods

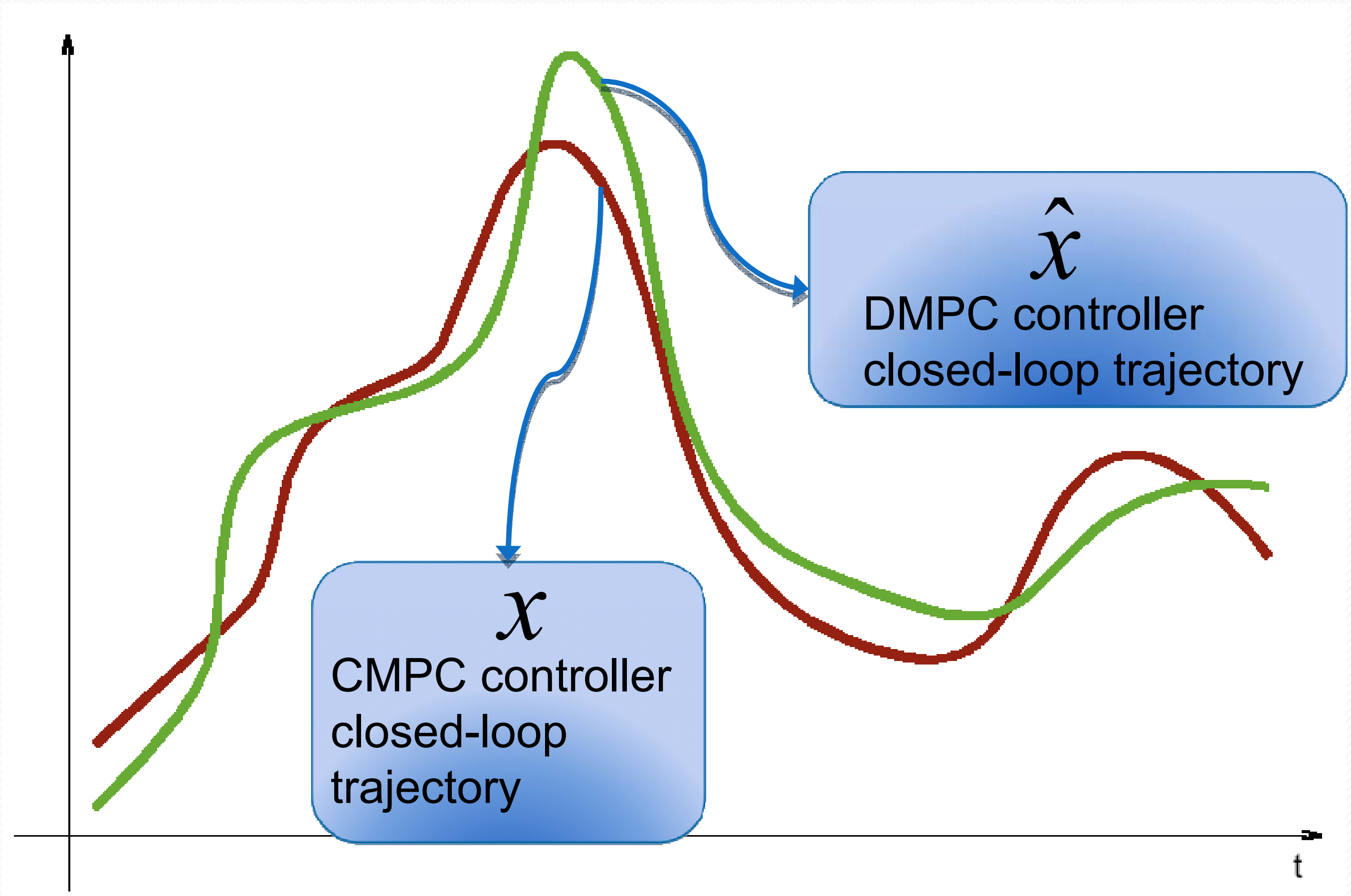
- Optimization approach

$$\min_{W_i, Z_i \; \forall i=1\ldots M} \left[\sum_{k=0}^{T-1} \| x(k+1) - \hat{x}(k+1) \|^2 \right]$$

- Sensitivity approach

$$S_{x_i x_j} = \frac{\partial x_i(k+1)}{\partial x_j} = a_{ij} \quad i, j = 1, \ldots, m$$
$$S_{x_i u_j} = \frac{\partial x_i(k+1)}{\partial u_j} = b_{ij} \quad i = 1, \ldots, m \quad j = 1, \ldots, n$$

Optimization approach (1/3)



Optimization approach (2/3)

$$\min_{\substack{W_i, Z_i \\ \forall i=1\ldots M}} \left[\sum_{k=0}^{T-1} \|x(k+1) - \hat{x}(k+1)\|^2 \right]$$

$$s.t. \quad \left[\begin{array}{c} y(k) = C\hat{x}(k) \\ u_i = f^i_{move}(y(k), Q(k:h)) \quad \forall i = 1\ldots M \\ \hat{u} = \sum_{i=1}^{M} Z_i u_i \\ \hat{x}(k+1) = A\hat{x}(k) + B\hat{u} + Q(k) \\ \\ W_{tot} = \left[\begin{array}{cccc} W_1 & W_2 & \cdots & W_M \end{array} \right] \quad ; \quad Z_{tot} = \left[\begin{array}{cccc} Z_1 & Z_2 & \cdots & Z_M \end{array} \right] \\ w \in W_{tot} \vee z \in Z_{tot} : w, z \in \mathbb{N} \wedge w, z \in [0,1] \\ \sum_{j=1}^{M \cdot m_i} w_{i,j} = 1 \quad \forall i = 1\ldots m \quad ; \quad \sum_{j=1}^{M \cdot n_i} z_{i,j} = 1 \quad \forall i = 1\ldots n \\ \sum_{i=1}^{m} w_{i,j} \leq 1 \quad \forall j = 1\ldots M \cdot m_i \quad ; \quad \sum_{i=1}^{n} z_{i,j} \leq 1 \quad \forall j = 1\ldots M \cdot n_i \end{array} \right]$$

$$f^i_{move}$$

$$u_i(k) = \boxed{\arg\min_{u} \left[\begin{array}{c} \sum_{h=0}^{T_d-1} x_i^T(h) W_i^T W^Q W_i x_i(h) + \hat{u}^T(h) Z_i^T W^R Z_i \hat{u}(h) \\ s.t. \quad \text{system 2.4} \end{array} \right]} \quad \forall i = 1\ldots M$$

Optimization approach (3/3)

Complexity Reduction
LQR controller

$$\min_{W_i, Z_i \quad \forall i=1\ldots M} \left[\sum_{k=0}^{T-1} \| x(k+1) - \hat{x}(k+1) \|^2 \right]$$

$s.t.$

$$\left\{ \begin{array}{c} \left(\begin{array}{c} P_i(k-1) = W_i^Q - A_i^T P_i(k) B_i (W_i^R + B_i^T P_i(k) B_i)^{-1} B_i^T P_i(k) A_i + A_i^T P_i(k) \\ F_i(k) = (B_i^T P_i(k+1) B_i + W_i^R)^{-1} B_i^T P_i(k+1) \\ u_i(k) = -F_i(k) x_i(k) \end{array} \right) \quad \forall i = 1 \ldots M \\ \hat{u}(k) = \sum_{i=1}^{M} Z_i u_i \\ \hat{x}(k+1) = A\hat{x}(k) + B\hat{u}(k) \end{array} \right\}$$

Sensitivity approach (1/3)

$$M_{tot} = [A \quad B]$$

$$m_{i,j} \in M_{tot}$$

Algorithm 1 Partition algorithm

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for all i among the rows **do**
    for all j among the rows **do**
        for all k among the columns **do**
            if $m_{i,k} = 1$ and $m_{j,k} = 1$ **then**
                for all c among the columns **do**
                    if $m_{i,c} = 1$ or $m_{j,c} = 1$ **then**
                        $m_{i,c} = 1$
                        $m_{j,c} = 1$
                    end if
                end for
            end if
        end for
    end for
end for

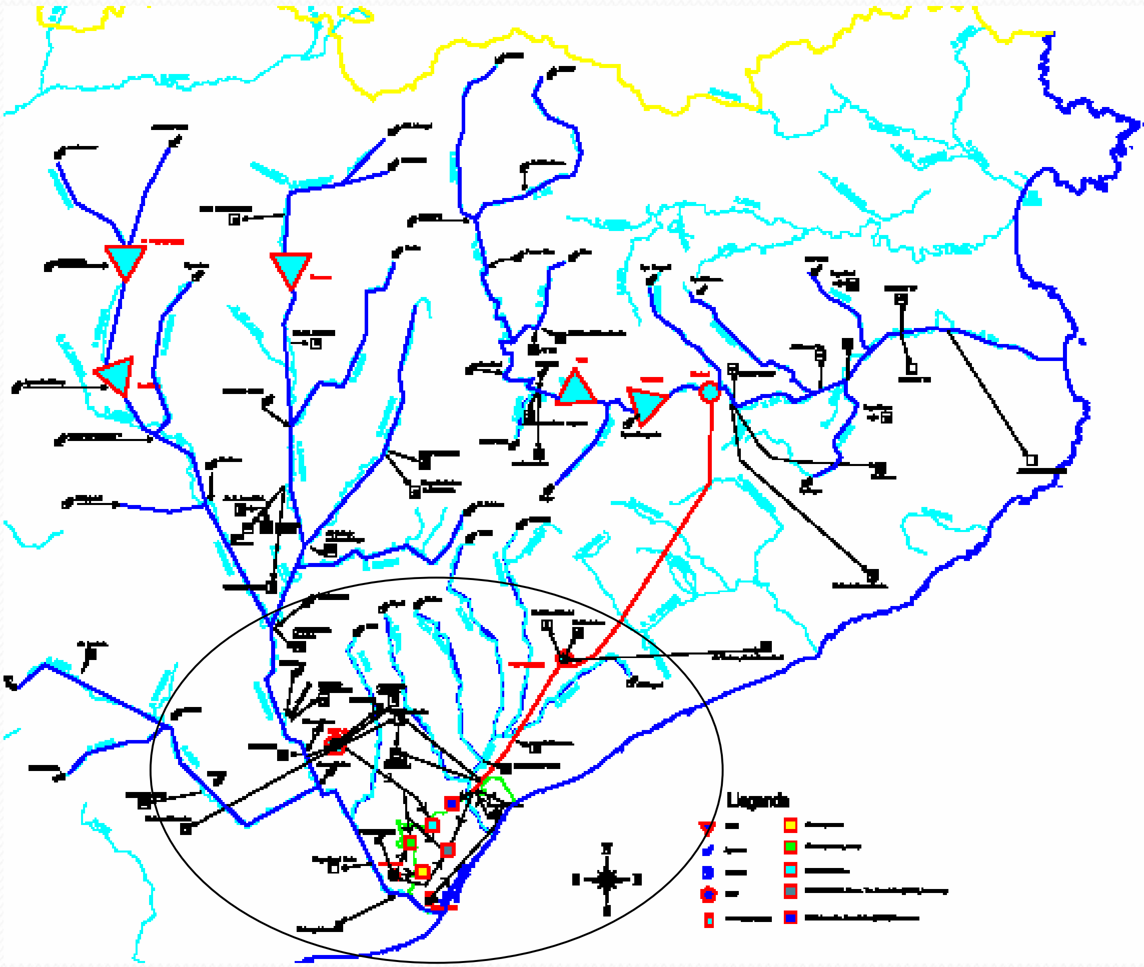
Sensitivity approach (2/3)

- Prefiltering
  - Magnitude
  - Correlation

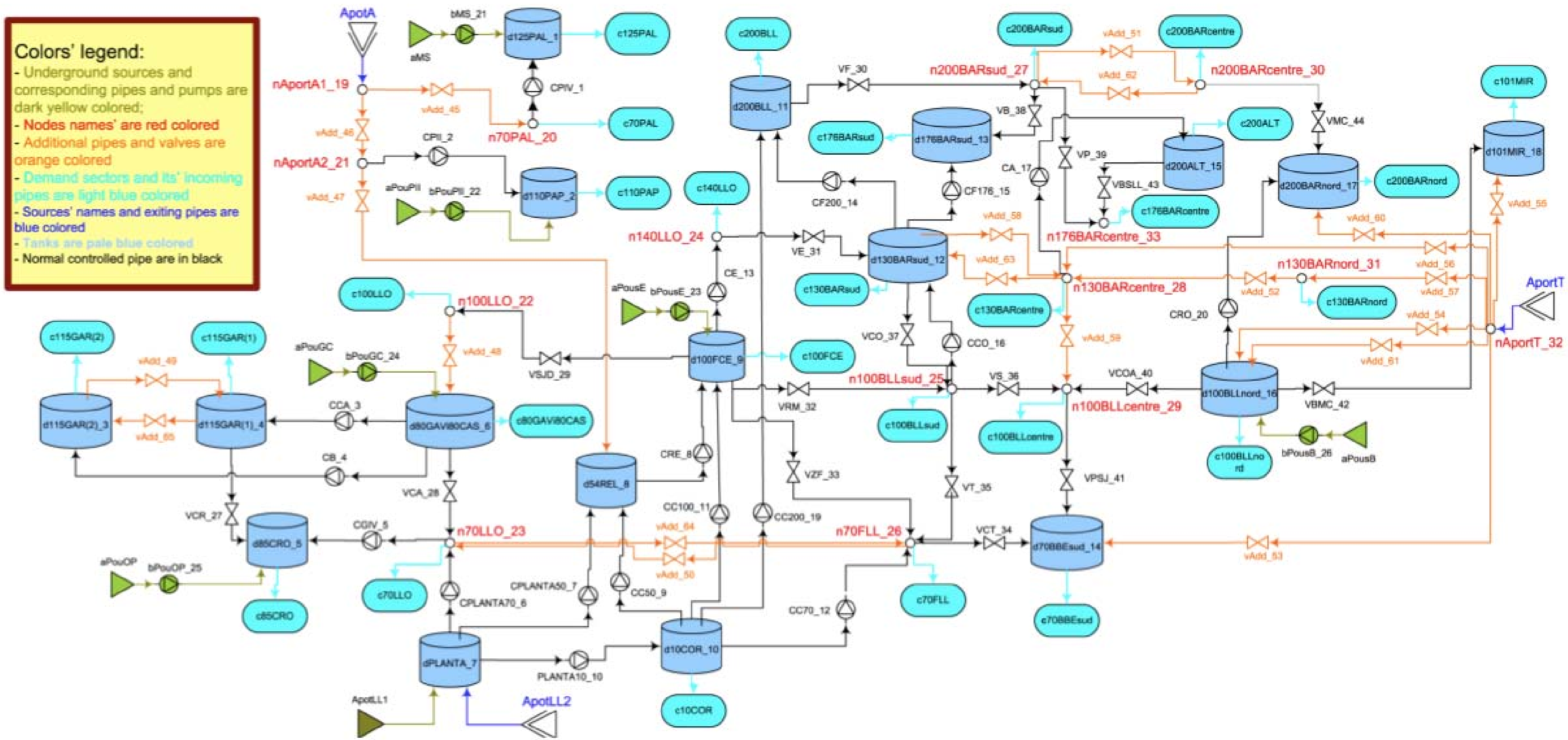
Sensitivity approach (3/3)

- Utility function
  - Element magnitude
  - Usage
  - Mixed

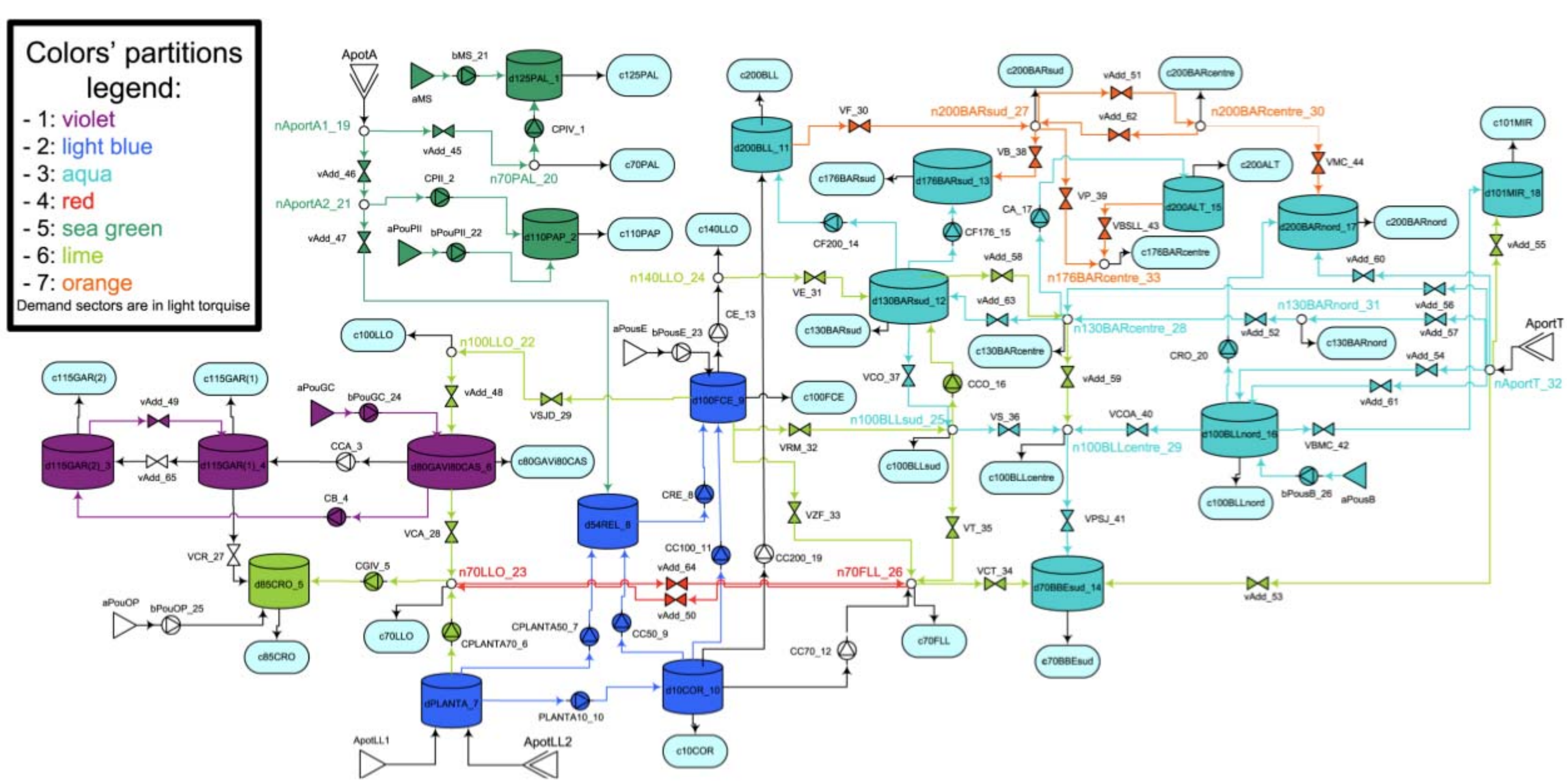
Barcelona water system



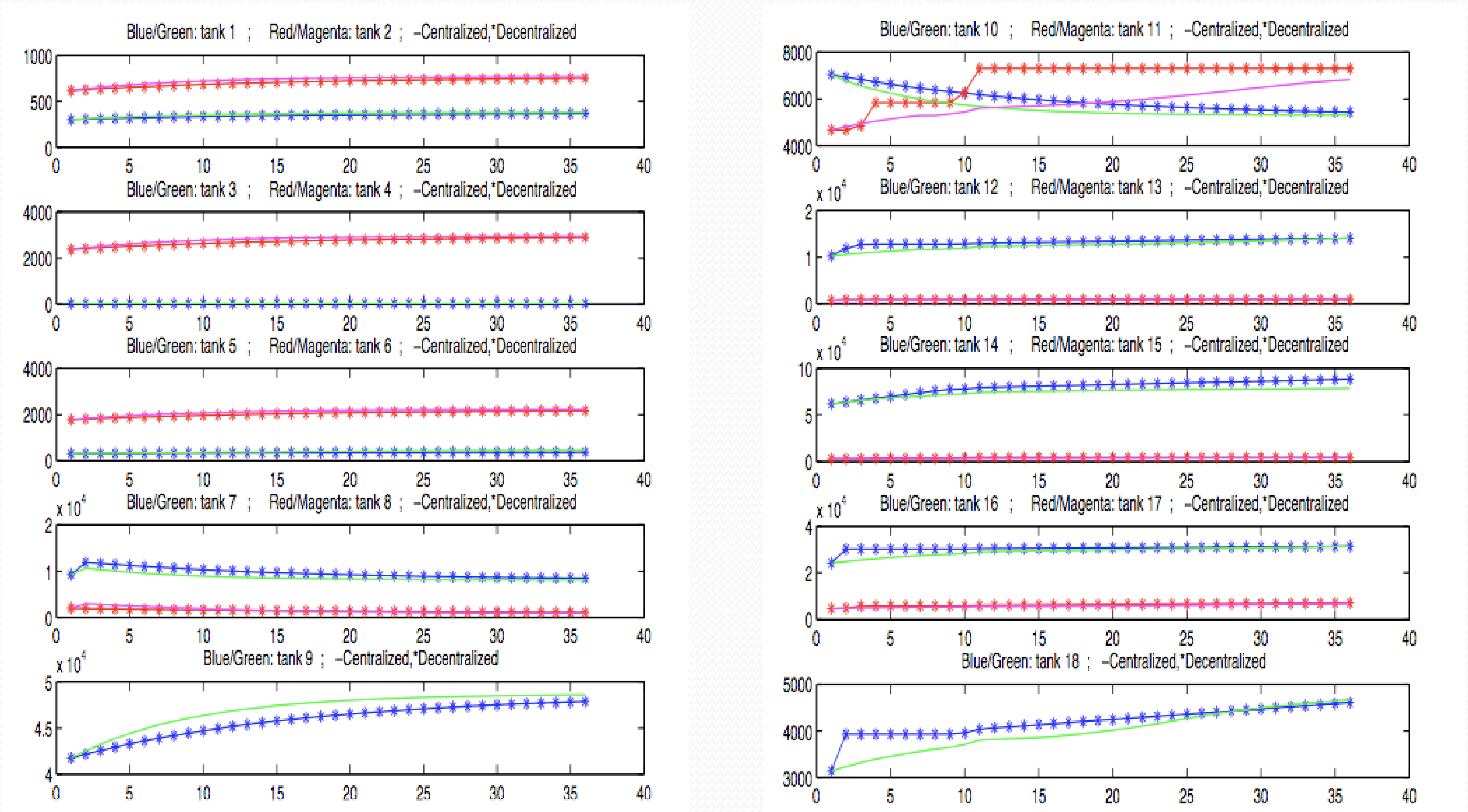
Barcelona DMPC case study (1/3)



Barcelona DMPC case study (2/3)



Barcelona DMPC case study (3/3)



Conclusions

- Previous experience/tools on centralized MPC of water networks have been presented.
- A Barcelona case study to be in the framework of the WIDE project is presented.
- A automatic partitioning algorithm to identify the subsystems of a large scale system has been presented.
- Preliminary results in the proposed Barcelona case study are promising.

Future works

- Propose the defined case study a the one to be used in the context of WIDE project trying to add/complete those aspects that make it more interesting
- Further validation of the partitioning algorithm
- Improvement of PLIO tool to include the partioning algorithm and to allow DMPC